# Gravitational Waves, Memory, and Astrometry

Abra Geiger and Eanna Flanagan
Cornell University

## Introduction

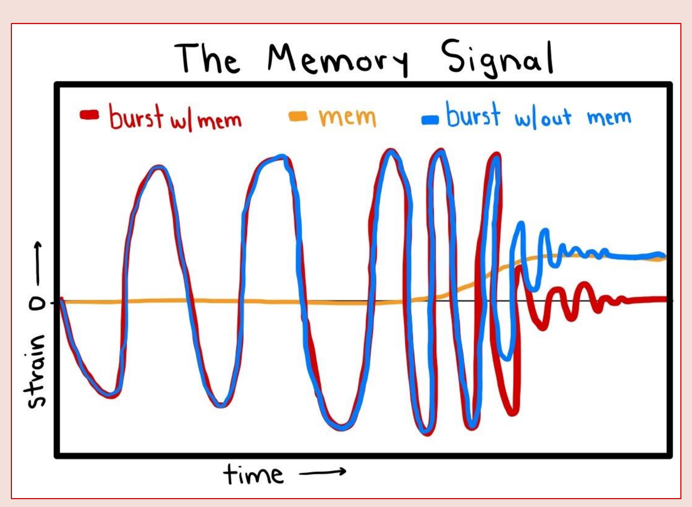
**This project strives to quantify and clarify the astrometric signal induced by gravitational waves and memory.**

**Gravitational waves** are ripples in the fabric of spacetime.

- Emerge from gravitational interactions such as black hole mergers throughout the history of the universe
- First detected in 2015
- Probe the universe in a new way, beyond limits of electromagnetic astronomy
- Inform about the wave source, the accuracy of our theory of gravity, and cosmology


**Astrometry** is a budding tool for gravitational wave detection.

- Precise measurement of the positions and motions of celestial objects
- Increasingly promising method as it becomes more precise
- Detects angular variations in the positions of objects on the sky induced by gravitational waves



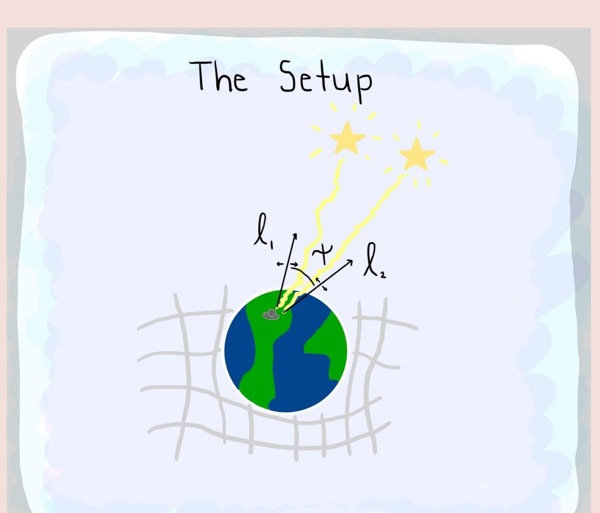

**Gravitational wave memory** is the lasting change in distance between test masses after the passage of a gravitational wave.

- Predicted by general relativity
- Not yet detected, but astrometry provides a promising future method of detection
- Arises from nonlinear interactions within gravitational waves themselves

## Methods

**We disagree with a proposed astrometric gravitational wave detection method in a recent *Nature* paper.**

**Their claim:** gravitational waves induce small variations in the binary angular separation for close pairs of stars, with the sensitivity to gravitational waves **scaling inversely** with the angle.[1]




**Our claim:** a term they claim to be order unity is actually proportional to the square of the binary angular separation, so the sensitivity to gravitational waves instead **scales linearly** with the binary angular separation.

**We quantify the astrometric signature of memory.**

Following published methodology, we have derived the spectrum of angular fluctuations for a burst with memory.[2]

## Results

**We disprove the potential for gravitational wave detection with stellar binary astrometry.**

**Deflection vector:**

$$\delta\ell^i(\tau_0,\ell) = \frac{\ell^i + p^i}{2(1+p\cdot\ell)} h_{jk}(0)\ell^j\ell^k - h^i_j(0)\ell^j$$

$\ell$: light source direction, $p$: gravitational wave momentum, $h_{ij}$: metric perturbation[3]

**Angle variation:**

$$\cos(\psi+\delta\psi) = g_{\mu\nu}(\tau,0)(\ell_1^\mu + \delta\ell_1^\nu)(\ell_2^\mu + \delta\ell_2^\nu)$$

$$\delta\psi = -\frac{\ell_1\cdot\delta\ell_2 + \ell_2\cdot\delta\ell_1 + h_{ij}\ell_1^i\ell_2^j}{\psi}$$

$$\delta\psi = -\psi\left(\frac{1}{2}h_{ij}v^iv^j - \frac{1}{2(1+p\cdot\ell_1)}h_{ij}\ell^i\ell^j - \frac{p\cdot v}{1+p\cdot\ell_1}h_{ij}\ell_1^iv^j + \frac{(p\cdot v)^2}{2(1+p\cdot\ell_1)^2}h_{ij}\ell_1^i\ell^j\right)$$

to first order in $h$ and $\epsilon = \psi$ with $\ell_2 = \ell_1 + \epsilon v + \varepsilon^2 u/2$

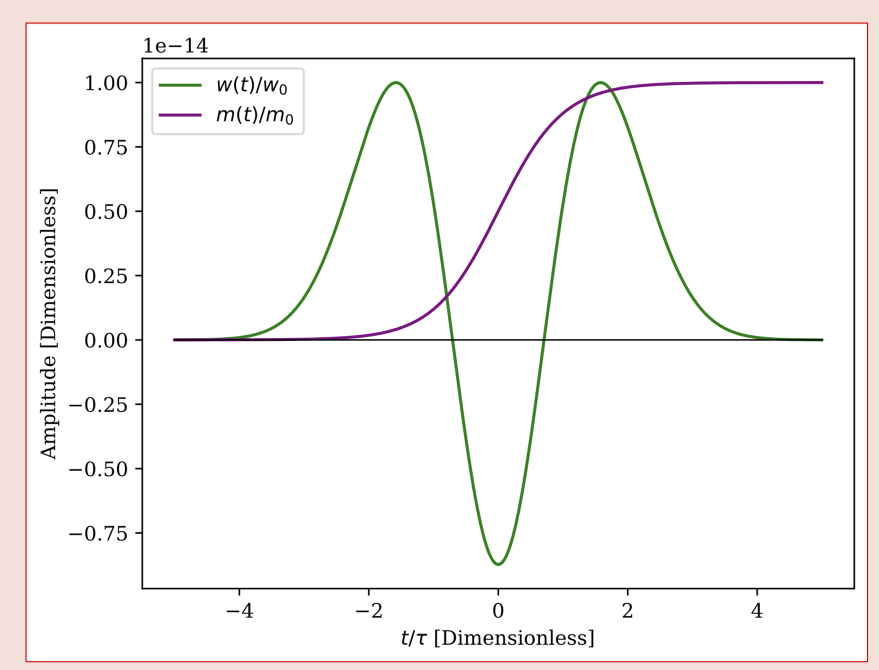
$\ell_{1,2}$: sources, $\psi$: angle between sources, $g_{\mu\nu}(\tau, x)$: metric at time τ and position x

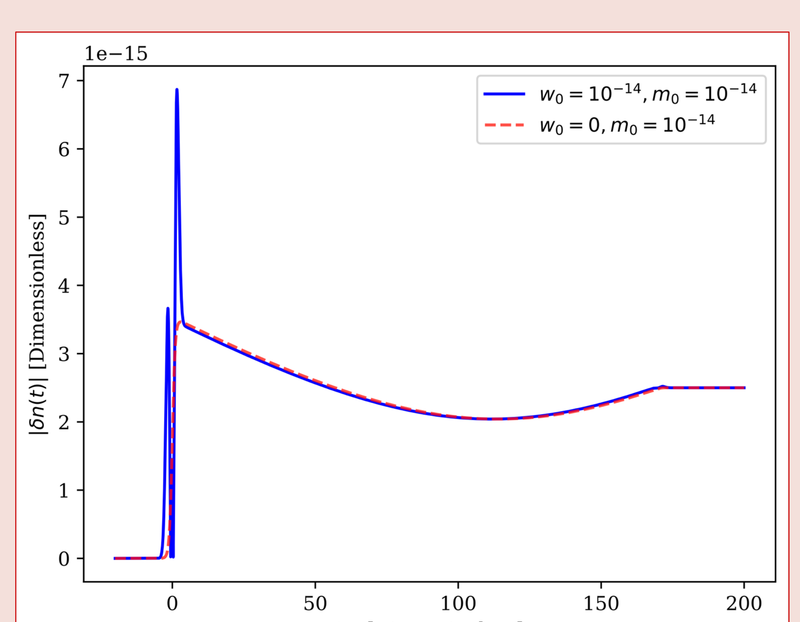
This shows that the **sensitivity to gravitational waves scales linearly with $\psi$.**

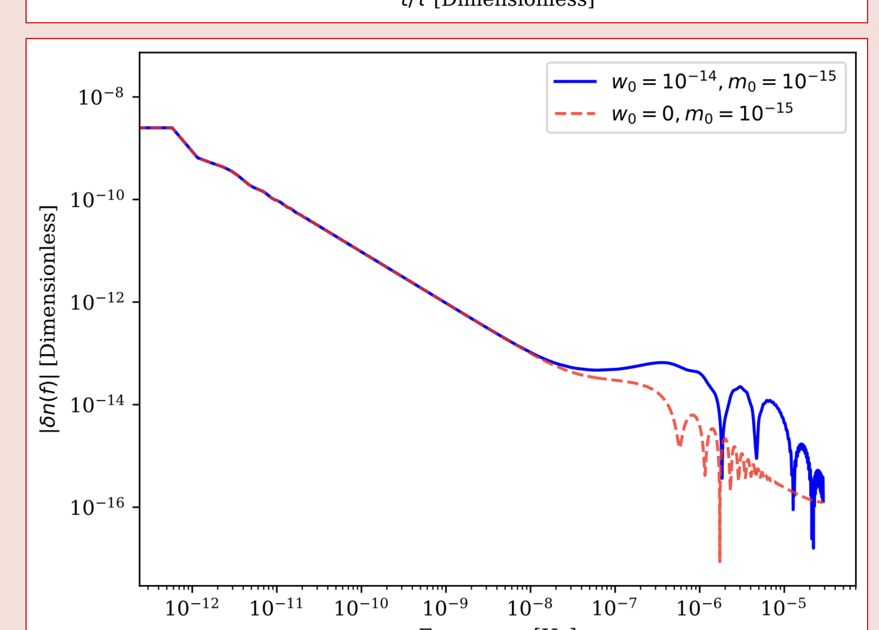
**We calculate the spectrum of angular fluctuations for a burst with memory.**

We have replicated the results of Madison 2020 to obtain the variation in the deflection vector with time for a localized burst of gravitational waves with memory, extending this to the spectrum of angular fluctuations.[2]

The left figure shows the gravitational wave burst, $w$, and memory, $m$, contributions.




Deflection vector timeseries (left) and spectrum (right) for a localized burst of gravitational waves with memory, specializing to a point in the Northern Hemisphere.

## Discussion & Conclusions

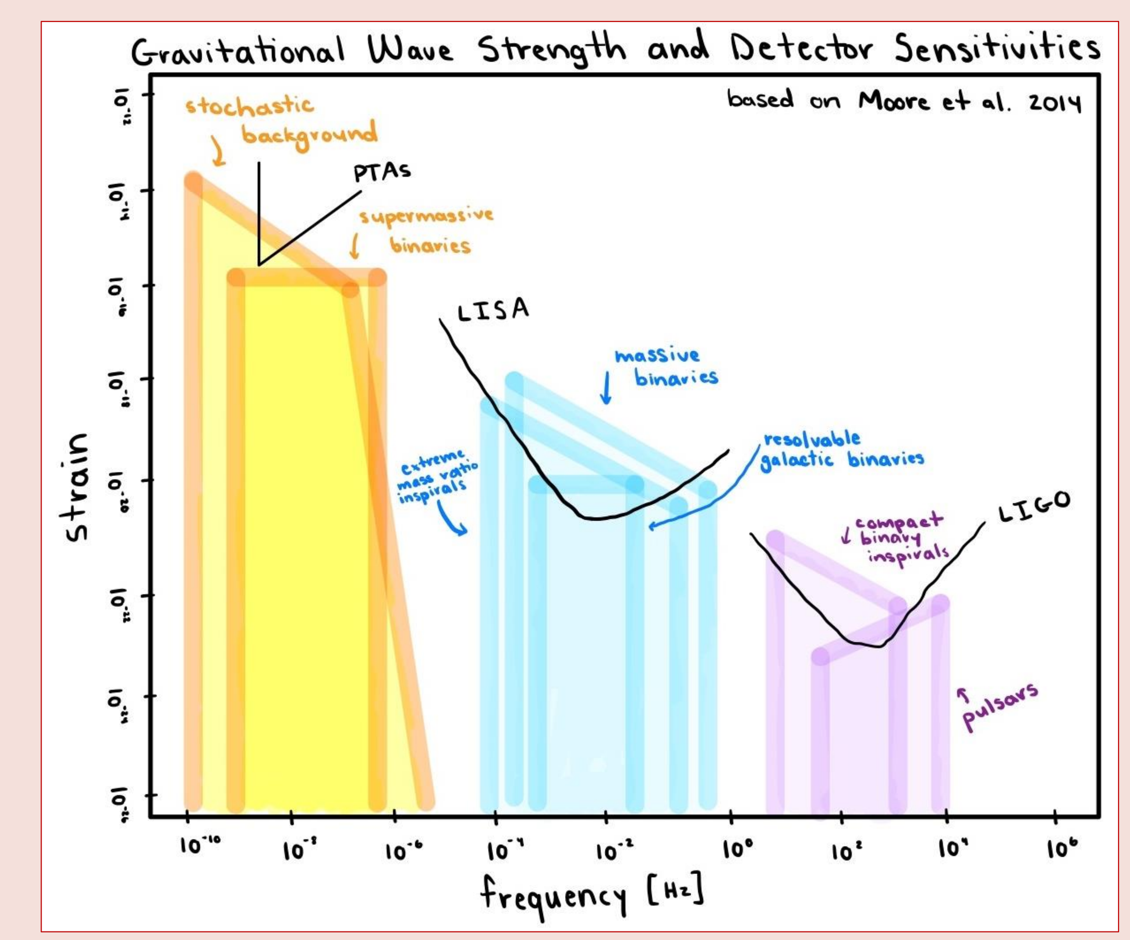
**Contrary to prior claims, sensitivity scales with angular separation, nullifying the feasibility of the proposed method.**

- In the limit of small $\psi$, the sources are coincident on the sky, so $\delta\psi$ vanishes.
- For the proposed binary separations of 1 to 10 milliarcseconds, the extra $\psi^2$ factor **reduces sensitivity by 15 to 17 orders of magnitude**.
- Detection of the astrometric signature induced by gravitational waves is still promising.
- It is ideal to detect the all-sky gravitational wave signal by observing many stars across the sky, rather than close pairs of stars.[3,4]

We are currently preparing a comment for submission to *Nature Matters Arising*.

**Better understanding gravitational wave memory and detecting it will influence our theories of gravity.**

- Memory provides a window into nonlinear general relativity
- We are now extending our work to a spectrum for a population of sources
- Goal is to **assess prospects for astrometric detection of memory**




## References & Acknowledgements

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